Solve the following differential equations. **SCORE: _____ / 30 PTS**

[a] $\dfrac{dy}{dx} = \dfrac{xy+3x-y-3}{xy-2x+4y-8}$ 8 PTS **FINAL ANSWER:** $Ce^{y-x} = \left(\dfrac{y+3}{x+4}\right)^5$

[b] $3(1+t^2)\dfrac{dy}{dt} = 2ty(y^3-1)$ 11 PTS **FINAL ANSWER:** $y = [C(1+t^2)+1]^{-\frac{1}{3}}$

[c] $(2y^2+3x)\,dx + 2xy\,dy = 0$ 11 PTS **FINAL ANSWER:** $x^2y^2 + x^3 = C$

EACH UNDERLINED ITEM (OR DESIGNATED ALTERNATE) IS WORTH 1 POINT

[a] $\dfrac{dy}{dx} = \dfrac{(x-1)(y+3)}{(x+4)(y-2)}$

$\displaystyle\int \frac{y-2}{y+3}\,dy = \int \frac{x-1}{x+4}\,dx$

$\displaystyle\int \left(1 - \frac{5}{y+3}\right) dy = \int \left(1 - \frac{5}{x+4}\right) dx$

$y - 5\ln|y+3| = x - 5\ln|x+4| + C$

$\dfrac{e^y}{(y+3)^5} = \dfrac{Ce^x}{(x+4)^5}$

$Ce^{y-x} = \left(\dfrac{y+3}{x+4}\right)^5$ OR $e^y(x+4)^5 = Ce^x(y+3)^5$

[b] $3(1+t^2)\dfrac{dy}{dt} + 2ty = 2ty^4$

$v = y^{-3} \rightarrow \dfrac{dv}{dt} = -3y^{-4}\dfrac{dy}{dt}$

$\dfrac{dy}{dt} = -\dfrac{1}{3}y^4\dfrac{dv}{dt}$

OR

$(1+t^2)v^{-\frac{4}{3}}\dfrac{dv}{dt} + 2tv^{-\frac{1}{3}} = 2tv^{-\frac{4}{3}}$

$-y^4(1+t^2)\dfrac{dv}{dt} + 2ty = 2ty^4$

$-(1+t^2)\dfrac{dv}{dt} + 2ty^{-3} = 2t$

$-(1+t^2)\dfrac{dv}{dt} + 2tv = 2t$

$\dfrac{dv}{dt} - \dfrac{2t}{1+t^2}v = \dfrac{-2t}{1+t^2}$

$\mu = e^{\int -\frac{2t}{1+t^2}dt} = e^{-\ln(1+t^2)} = \dfrac{1}{1+t^2}$

$\dfrac{1}{1+t^2}\dfrac{dv}{dt} - \dfrac{2t}{(1+t^2)^2}v = \dfrac{-2t}{(1+t^2)^2}$

CHECKPOINT: $\dfrac{d}{dt}\dfrac{1}{1+t^2} = -(1+t^2)^{-2}(2t)$

$= \dfrac{-2t}{(1+t^2)^2}$ ✓

$\dfrac{1}{1+t^2}v = \displaystyle\int \frac{-2t}{(1+t^2)^2}\,dt + C$

$= \dfrac{1}{1+t^2} + C$

$v = 1 + C(1+t^2)$

$y^{-3} = C(1+t^2) + 1$

$y = [C(1+t^2)+1]^{-\frac{1}{3}}$

[c] $\underbrace{(2y^2+3x)}_{M}dx + \underbrace{2xy}_{N}dy = 0$

$$\frac{M_y - N_x}{N} = \frac{4y - 2y}{2xy} = \frac{2y}{2xy} = \frac{1}{x}$$

$$\mu = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x$$

$$\underbrace{(2xy^2+3x^2)}_{M}dx + \underbrace{2x^2y}_{N}dy = 0$$

CHECKPOINT: $M_y = 4xy = N_x$ ✓

$$f = \int 2x^2y\,dy$$

$$= x^2y^2 + C(x) \qquad \text{OR} \qquad x^2y^2 + x^3 + C(y)$$

$$f_x = 2xy^2 + C'(x) = 2xy^2 + 3x^2 \qquad \text{OR} \qquad 2x^2y + C'(y) = 2x^2y$$

$$C'(x) = 3x^2$$

$$C(x) = x^3 \qquad \text{OR} \qquad C(y) = 0$$

$$f = x^2y^2 + x^3 = C$$